Environmental Fluid Dynamics, Exercise Sheet 6
Christoph Garbe and Jana Schnieders
Wintersemester 2012/2013

To be returned until 21.12.2012, in the lecture, by mail to jana.schnieders@iwr.uni-heidelberg.de or to Speyerer Str. 6, 3rd floor (G 302 or H 306)

1 (30 Pts)
Starting with the RANS (Reynolds averaged Navier-Stokes) momentum equation, derive the equation for the rate of change of the kinetic energy of the average flow field (Kundu, equation 12.46).

\[
\frac{\partial E}{\partial t} + \frac{\partial E}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{U_i}{P_0} + 2\nu S_{ij} - \frac{\partial U_i}{\partial x_j} \right) - 2\nu S_{ij} S_{ij} - 2\frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j} - \frac{8}{P_0} \pi I_3,
\]

for \( E \) the mean kinetic energy and a gain of turbulent kinetic energy. It is commonly known as the shear production term.

2 (30 Pts)
From the equation for the kinetic energy of the average flow field (s.a.) a production of turbulent kinetic energy \( P \) can be defined as

\[
P = -u'_i u'_j \frac{\partial U_i}{\partial x_j}
\]

with the Reynolds stresses \( u'_i u'_j \) and the mean-flow gradients \( \partial U_i/\partial x_j \).

2.1
Use the eddy-viscosity assumption (the momentum transfer caused by turbulent eddies can be modeled with an eddy viscosity) and obtain an expression for \( P \). How does this relate to dissipation?

2.2
Show that the production is only affected by

1. the symmetric part of the velocity-gradient tensor, and

2. the anisotropic part the of Reynolds-stress tensor.

1