Validation of Eddy-renewal model by numerical simulation

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Abstract. The eddy-renewal model proposes that the dominant vortical flows of the near-surface turbulence can be approximated by pairs of stationary, spatially periodic, along-wind eddies. The upwellings and downwellings induced by eddies are associated with the elongated warm patches and cold streaks observed in the surface infrared images, respectively. A direct numerical simulation of a wind-driven aqueous turbulent boundary layer shows that the formation of along-wind vortex pairs strongly relate to the temperature structures which is consistent with the concept of the model. A reliable bulk temperature is predicted by the model for known surface temperatures resulting from the simulation; however, the estimated heat flux is higher by a factor of two. This overestimation of the heat flux may be due to the fact that the turbulent-convection effect on transport has the same order of magnitude as the diffusivity effect in the thin diffusive sublayer beneath surface.
Key Words: eddy-renewal model, numerical simulation, upwelling, estimated heat flux, bulk temperature.

1. Introduction

Gas transfer across the air-sea interface has gained significant attention in geophysical and meteorological research in the last decade. Remarkable progresses in the experimental techniques make it possible to gain realistic transfer quantities highly resolved to help us analyze the processes in detail. However, the physical mechanisms of transfer processes in the mass boundary layer are still not fully understood, mainly because of complicated dynamics caused by interactions between wind, wave, and turbulence. Statistical models such as the surface-renewal model are acceptable for estimating transfer velocities (Garbe *et al*. 2004). However, these types of models lack in linking the hydrodynamics and transfer behaviors in the mass boundary layer.

Fortescue and Pearson (1967) suggested that the mass transfer across interface in a turbulent channel flow is controlled by the large scale turbulent eddies. They assumed there to be no turbulence-generating mechanism at the surface due to shear flow but turbulence is generated by a square-mesh oscillating grid. Hence, the general eddy pattern will not be substantially different from near-surface region to the bulk field. The eddies are taken to be square, and the velocity field of eddies are treated as a two-dimensional, roll cells which rotate in the spanwise direction with a sinusoidal distribution in both streamwise and vertical directions. Two horizontal boundaries of one-half eddy are corresponding to a upwelling and downwelling respectively. The length scale of eddies can be represented in the integral length scale and the scale of velocity field is taken as twice of root-mean square turbulent intensity. The turbulent intensity is decided by the known velocity from the size of inserted square grids which made from circular cylindrical rods and the size of rods. The transfer coefficient then can be predicted from solving a steady, two-dimensional convection-diffusion equation. It is shown that their experimental measurement in mass transfer coefficient is about 10-15 percent below the value predicted by the model. They conclude that "it is supposed that only 70 percent of the total turbulent intensity resided in the large model eddies."

Lamont and Scott (1970) considered the mass transfer in a bubbles-transported, turbulent pipe flow and proposed that the small scales of turbulent motion should be more efficient than the large ones for mass transfer across an interface. An illustration of their idea is that it might have the much smaller scale eddies superimposed on the major eddy in the vicinity of interface, which should mainly dominate transfer if their intensity is sufficiently large. Therefore, they suggested the smallest scale eddies should be more efficient for the mixing and may control the overall transfer rate. Similar to Fortescue and Pearson (1967), they assumed that these idealized, viscous eddies can be represented as the steady, sinusoidal shearing motions, and consequently, they presented the dependence of non-dimensional transfer coefficient and Peclet number by resolving the convection-diffusion equation. The transfer coefficient is normalized by the length scale of eddies and molecular diffusivity, and the Peclet number is defined as the length scale of eddies multiplied with the velocity scale divided by molecular diffusivity. The universal function of the turbulent kinetic energy spectrum for well-developed turbulence is applied to determine the velocity scale. In consequence of replacing the velocity scale to the turbulent kinetic energy dissipation rate, transfer coefficient can be represented as a function of molecular diffusivity, energy dissipation rate and length scale. The result shows the dependence of wave number with transfer coefficient is exponent of 0.33, which means, it grows very slowly with the increased wave number in the moderate to high wave number region and do not have any significant contribution from the smallest eddies. They concluded that "the inertial subrange cannot be ignored as was initially postulated." Even though their initial assumptions failed, they assumed the transfer behavior of the inertial motions is similar to the small viscous motions and estimate the overall transfer coefficient by integrating for the most concerned range of lengthscale. It is supposed to include all the scales smaller than the dimensions of a particle or a bubble, and as a result the range from 13.6η to 0. 68η was suggested, where η is Kolmogorov microscale, which also indicated that parts of the inertial subrange are also involved in the estimation of an overall transfer coefficient. Two comparisons of predicted overall transfer coefficient with experimental results were carried out. In the turbulent bubble pipe flow, the modeling prediction shows the dependence of Reynolds number with transfer coefficient is exponent of 0.69, however, the experimental data is exponent of 0. 52. Besides, by applying the experimental data of turbulent channel flow from Fortescue and Pearson (1967) to compare with prediction of present modeling, the agreement is not as good as the model of Fortescue and Pearson (1967).

Both of these two papers attempted to build up a link between turbulent behavior and mass transfer across the interface under a hydrodynamically realistic concept. Nevertheless, both of them assumed there are no turbulence generated-mechanism on the interface and no consideration of convection due to the mean flow. However, on the "free" air-water interface, the major turbulent production is generated from the surface due to the momentum transport from the wind-blowing in the air. Moreover, the characteristics of turbulence generated from the water surface should be different with the turbulent generated from the bottom or well-developed, isotropic turbulence which is far away from any boundary. Besides, the short gravity-capillary waves which induce a vortex-strength boundary layer in the vicinity of surface might also have some important impacts in the transfer processes. Hence, these reasons make the applicability of the models to the transfer processes at the air-sea interface questionable.

By observing the infrared images of air-water interface in both field and laboratory experiments, typical surface thermal structures are the elongated patches of warm water accompanying with the alternating cold streaks paralleled with wind-blowing direction in the low to moderate wind speeds. Hara *et al.* (2007) proposed the eddy-renewal model which describes that, underneath these surface signatures, the distinct roller type of turbulence patterns aligned in the wind direction should illustrate the scale transfer across wind-driven surface. It is assumed that the depth of the diffusive sublayer is very thin in comparison with the length scale of eddies, and the convection effect due to the turbulent eddies has the same order of magnitude with the diffusion. A theoretical surface temperature distribution of an eddy by resolving the stationary convection-diffusion equation is applied to estimate the bulk temperature for surface infrared images from field and laboratory experiments by fitting and suitable rescaling. The concept of the eddy cell motion which are corresponded to the upwellings and downwellings emerged in the eddy-renewal model are similar with the model of Fortescue and Pearson (1967) and the model of Lamont and Scott (1970). However, it is the first time to connect the eddy cell motion with the observed thermal structures on the air-water interface.

A numerical modeling of a wind-driven, aqueous turbulent boundary-layer flow was developed by Tsai and Hung (2007). All the major features of a wind wave, including the generation of parasitic capillaries ripples and the formation of along-wind vortex pairs associated with the nonlinear interaction between the surface wind wave and the shear flow are observed in the simulation results. In this study, we carry out a simulation to study the scalar transfer processes across wind-wave surface. The heat is treated as a passive tracer and temperature field is resolved by solving a convection-diffusion transport equation with instant velocity field simultaneously. The temperature structures will be connected with the turbulent structures to infer the transfer mechanism and the "known" underlying turbulent structures will be applied to demonstrate the applicability of eddy-renewal model.

2. Observation of along-wind vortex pairs in numerical simulation

A direct numerical simulation of a wind-driven, aqueous turbulent boundary-layer flow is carried out. This mathematic modeling resolves an incompressible and Newtonian flow field which is governed by the Navier-Stokes and the continuity equation. The simulation domain is bounded by the periodic boundaries in the horizontal directions, a fully-nonlinear free-surface on the top, and a free-slip boundary at the bottom. The evolution of a gravity-capillary wave in wavelength 7.5 cm with the initial steepness 0.25 is performed. A normal wind forcing with an average magnitude of 0.6 dyn/cm^2 and a wind-shear stress with an average magnitude of 1.64 dyn/cm^2 act on the surface continuously. We assume that these forces approximately represent the momentum input from air into water on the wind speed 4.8 cm/s at the height of 20 cm. The details of mathematic formulation and numerical methods for simulation are presented in Tsai and Hung (2007). It is commonly accepted in the field and laboratory experiments that heat can be used as a passive tracer in studying transfer processes across air-water interface. The temperature field is governed by the convection-diffusion transport equation is solved with the instant velocity field simultaneously. The transport equation is given by:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\kappa\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right], \tag{1}$$

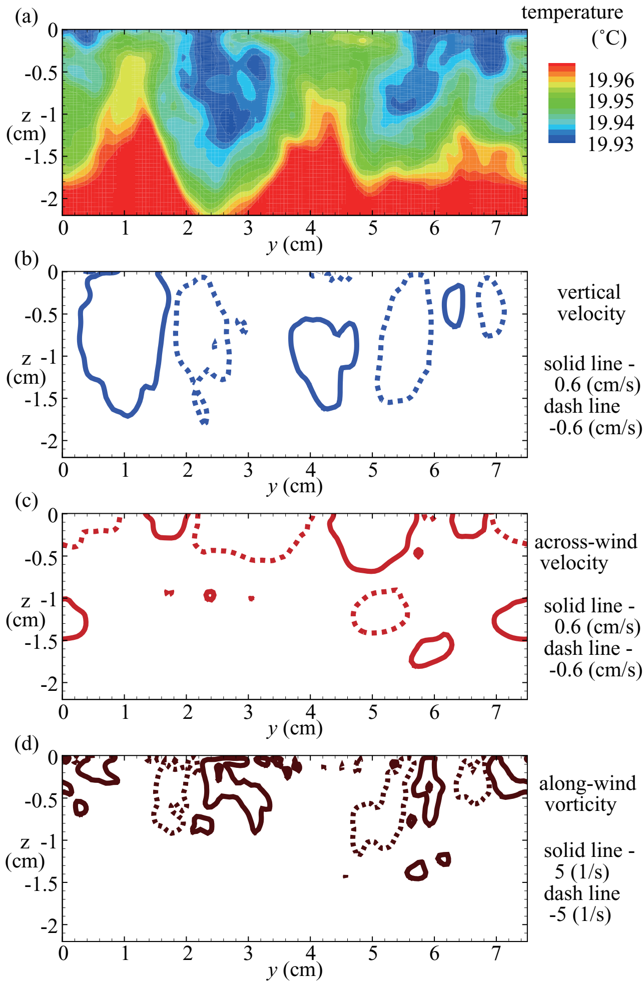

Figure 1 The quasi-stationary temperature structures and velocity structures for a gravity-capillary wave of wavelength 7.5 cm by taking average in the along-wind direction. (a) temperature field, (b) vertical velocity, (c) across-wind velocity, (d) along-wind vorticity. A reference temperature is set as 20 °C at the bottom of simulation domain, and a related heat flux at surface is set as 0.0065 W/cm^2. The upwellings located at y$\approx$1 cm and y$\approx$4.2 cm are associated with the warm water raised up from bottom toward surface. The horizontal positive and negative velocities tend to push away the cool water on surface in the opposite directions as a divergence zone in the associated vicinity of surface. The occurrence of the induced along-wind vortex pairs is consistent with the concept of eddy-renewal model.

where $T(x, y, z, t)$ is temperature field; κ is the thermal diffusivity; x, y, and z are along-wind, cross-wind and vertical direction, respectively; and u, v, and w are represented as velocity components in x, y, and z directions, respectively. Note that the velocity field is represented by the summation of the effects from the orbital motions of gravity-capillary wave, nonlinear wave-wave interaction, shear flow induced by wind and their interactions with turbulence.

By considering the general case that the evaporation and emission on the air side always take the energy away from the water side, the temperature at surface is always cooler than in the bulk. Therefore, the heat flux on surface is assumed as to be a positive constant according to the following definition as our boundary condition:

$$j_H = -k\frac{\partial T}{\partial z} = -\kappa\rho C_p\frac{\partial T}{\partial z}, \tag{2}$$

where j_H is the heat flux across surface; k is the thermal conductivity; ρ is water density, C_p is the specific heat. At the bottom boundary, we assume it is a uniform distribution of temperature field.

The simulation result shows the spatial correlated structures between surface turbulence and temperature distributions on surface. Since we observed the elongated cool streaks and warm patchs in the wind direction in the surface temperature structures, we attempt to find the relationship between the temperature and the velocity field in the point of view of taking average in the along-wind direction to focus our interest of the spatial variation of these quantities in the cross-wind direction.

From the averaged two dimensional fields, we observe that significant vortex pairs appear in the wind-driven boundary layer. These vortex pairs enlarge during the evolution and reach a quasi-stationary state in the terminal stage of simulation. The corresponding upwellings and downwellings induced by the vortex pairs exist strong correlations with the temperature structures as shown in Figure 1. The strong upwelling flow (i.e. positive, vertical velocity) which is located at $y \approx 1$ cm and $y \approx 4.2$ cm are consistent with locations at which the warm water rises up from bottom towards the surface. The horizontal positive and negative velocities tend to push away the cool water on the surface in the opposite directions as a divergence zone in the associated vicinity of the surface. This indicates that the temperature field is significantly dominated by convection effects due to the flow field.

3. Eddy-renewal model

The basic concept of the eddy-renewal model is that the near surface turbulence is assumed as two-dimensional, pairs of stationary eddies which are spatially periodic and orientating in the wind direction. Therefore, the transport equation can be simplified as follows:

$$v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\kappa\left[\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right], \tag{3}$$

Here, the boundary condition of temperature at surface is assumed to have a constant heat flux and the bottom boundary is assumed to be a constant value as bulk temperature. Furthermore, it assumes the horizontal velocity of eddies can be represented as a sinusoidal profile at surface and the Taylor expansion in depth is applied for the vertical variation as following:

$$v=v_0\sin(k_e y)+\left[\frac{\partial v}{\partial z}\right]_{z=0}z+\dots. \tag{4}$$

where k_e is the wavenumber of the eddy; $k_e=2\pi/\lambda_e$, which is defined by the length of the eddy λ_e. Because it is required to satisfy the continuity, the vertical velocity can be represented as

$$w=-zk_e v_0\cos(k_e y)-\left[\frac{\partial^2 v}{\partial z\partial y}\right]_{z=0}\frac{z^2}{2}+\dots. \tag{5}$$

To non-dimensionalize the transport equation, the variables are represented as non-dimensional ones as below:

$$\tilde{y}=k_e y,\ \tilde{z}=\frac{z}{\delta},\ \tilde{T}=\frac{(T-T_b)k}{\delta\cdot j_H},\quad \text{where } \delta^2=\frac{\kappa}{k_e v_0}.$$

Here, the turbulent-convection effect is assumed to have the same order of magnitude as heat diffusion in the vertical direction. The depth of the diffusive sublayer δ is given by the diffusivity κ and the surface-divergence scale $k_e v_0$. Consequently, the non-dimensional transport equation and boundary conditions can be expressed as

$$\sin\tilde{y}\frac{\partial\tilde{T}}{\partial\tilde{y}}-\tilde{z}\cos\tilde{y}\frac{\partial\tilde{T}}{\partial\tilde{z}}=\frac{\partial^2\tilde{T}}{\partial\tilde{z}^2}+O(k_e\delta), \tag{6}$$

$$\frac{\partial\tilde{T}}{\partial\tilde{z}}=-1,\quad \text{at } z=0, \tag{7}$$

$$\tilde{T}=0,\quad \text{at } z=-\infty. \tag{8}$$

Here, it is assumed that the depth of the diffusive sublayer is much smaller than the wavelength of the eddies ($k_e\delta\ll 1$), and the transport equation is described as a heat diffusion governed by a single set of governing equations and boundary conditions

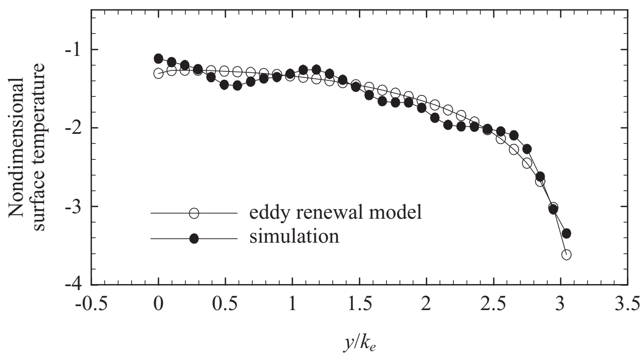

Figure 2 Fitting the surface temperature of a one-half of eddy detected from simulation result with the solution of eddy-renewal model. k_e is the wavenumber of the eddy.

without the consideration of the size and intensity of the turbulent eddies (Hara *et al*. 2007). The model provides a universal, theoretical surface temperature profile of eddies, and it was shown to be comparable with the actual surface temperature profiles which are measured in both laboratory and field experiments by suitable rescaling (Hara *et al*. 2007).

4. Results and discussion

4.1 Estimation of bulk temperature

A theoretical spatial variation of surface temperature for an eddy is provided by the eddy-renewal model. We attempt to estimate the bulk temperature of the simulated temperature by fitting the theoretical profiles with the simulated ones by

$$\tilde{T}=(T_{\mathrm{sf}}-T_{\mathrm{bulk}})\cdot C_f, \qquad (9)$$

where $\tilde{T}$ is the theoretical solution from the model which is a non-dimensional surface temperature profile; T_{sf} is simulated surface temperature for one eddy with dimension; T_{bulk} is the bulk temperature with dimension which is the variable we are interested in, and C_f is a scaling parameter. In this formula, T_{bulk} and C_f are set as the free parameters for fitting. The least square method is applied to achieve the best fitted. Because there are several eddies occurred in our simulated domain as shown in top panel of Figure 1, we should detect the eddy as our first step. Here, we define a one-half of the eddy is bounded between the local maximum and local minimum. If the oscillations in the surface temperature profile make detecting of eddies erroneous, then, the zero-crossing points in the cross-wind velocity profiles are used as a secondary criterion. Figure 2 shows the fitting between theoretical

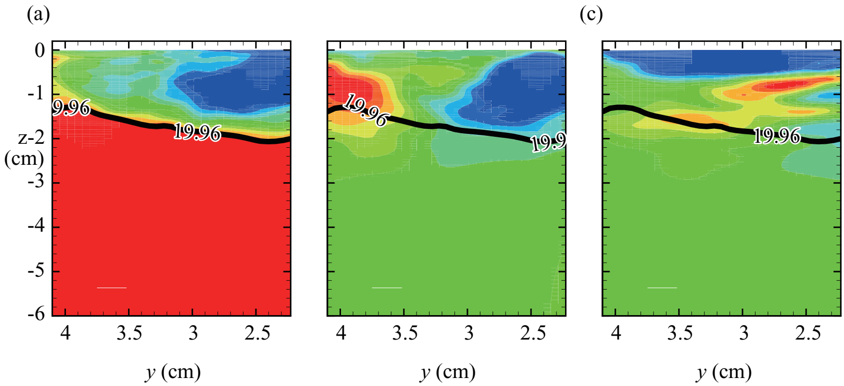

Figure 3 Contour line for bulk temperature in (a) temperature field, (b) cross-wind velocity field and (c) vertical velocity field. The scale of flood contour in temperature is maximum value 19.96 (ºC) as red color and minimum value 19.93(ºC) as blue color. The scale in velocity is maximum value 0.6 (cm/s) as red color and the minimum value −0.6 (cm/s) as blue color. The left boundary in this one-half eddy is corresponded to the upwelling in the model, and the right boundary is corresponded to the downwelling.

solution and simulation data for a one-half of eddy and the bulk temperature will be decided while the best fitted appears.

The bulk temperature estimated by the model is drawn as a contour line in the temperature field and velocity fields in Figure 3. For an one-half eddy detected from simulation data which is located between y=2.23 cm and y=4.10 cm in Figure 1, the estimated bulk temperature is 19.96 ºC for the maximum value 19.94 ºC and minimum value 19.91 ºC in surface temperature profile. For this one-half eddy, the upwelling is located at y=4.10 cm, and the downwelling is located at y=2.23 cm. The bulk temperature is found in 1.4 cm below the surface in the upwelling region and 1.9cm below in the downwelling region. These are further away from the interface than generally expected. However, it appears that most of the active turbulence is bounded between the 'bulk' line and water surface as we show in Figure 3 (b) and (c).

4.2 Estimation of heat flux

In the last section, we proposed a method to estimate the bulk temperature for a known temperature profile by fitting to a theoretical, universal surface temperature profile. If we have additional information of the magnitude of the cross-wind velocity, we would be able to predict the temperature field by resolving the governing equation of the model with boundary conditions as known bulk temperature in the bottom of eddy and known surface temperature profile on the top. The detection of surface temperature and the following estimation of bulk

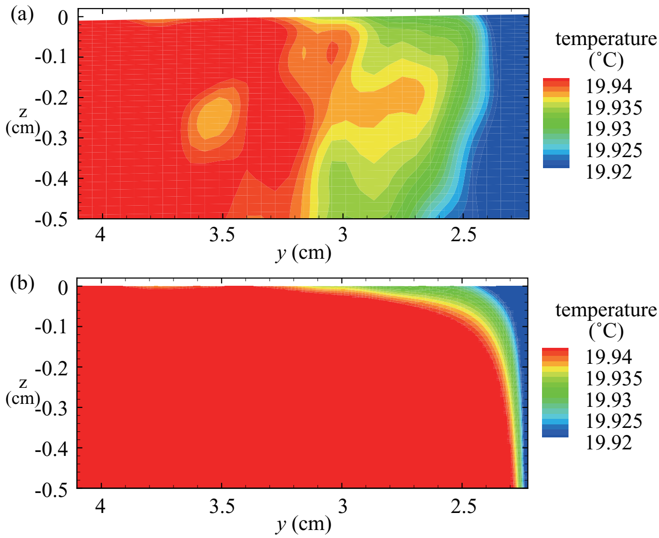

Figure 4 Temperature field for one-half of eddy. (a) detected from simulation result. (b) predicted by eddy-renewal model. The left boundary in this one-half of eddy is corresponded to the location of upwelling, and the right boundary is corresponded to the location of downwelling.

temperature are as described in the last section. The formula (9) is applied to non-dimensionalize surface temperature and bulk temperature. The predicted temperature field is the solution of the governing equation (6) with bottom condition (8) and another Dirichlet boundary condition on the top, which is represented as

$$\tilde{T}=\tilde{T}_{sf}, \quad \text{at } z=0 . \tag{10}$$

Here, the length scale in the vertical direction δ is decided by known parameters, which are the diffusivity κ and the surface-divergence scale $k_e v_0$. Additionally, one boundary condition in the horizontal direction is required. Here, we assume the temperature on the upwelling to be equal to the bulk temperature. A comparison of the simulated temperature field and the predicted one is shown in Figure 4. It reveals that the convection-effect in the model is much weaker compared to the simulated result. The simulated result show the convection-effect almost dominates the whole range from surface down to the depth of 0.5 cm, however, the diffusion-effect dominates this area in the model-predicted result. Furthermore, we calculate the vertical gradient of temperature on surface to infer the heat flux which is we are interested in. The result indicates the estimated heat flux is higher with a factor 2 as shown in Figure 5. This overestimated heat flux can be related to

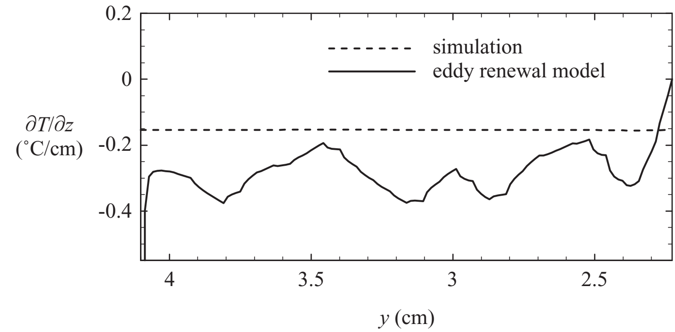

Figure 5 Spatial variation in vertical gradient of surface temperature on a one-half of eddy. Solid line: estimated by eddy-renewal model; dash line: simulation result.

underestimated convection-effect on the transport problem. As shown in Figure 4, the temperature field is strong convected from both the two horizontal boundaries and also from the surface, but this is not observed in the model-predicted result. This means that the field underneath surface is more strongly mixed than expected by model. This might refer to the assumption of the eddy-renewal model which is that the turbulent-convection effect has the same order of magnitude as heat diffusion in vertical direction. However, the turbulent-convection effect should dominate in the transport processes. Besides, we can conclude that eddies should have same length scale in both horizontal and vertical direction according to the turbulent velocity-structures as shown in Figure 1. Therefore, the model supposes that eddies exist in a very thin diffusive sublayer, which is not consistent with what we observed in the simulation result.

5. Conclusions

For water-side controlled transfer processes across the air-water interface, in principle it is feasible to deduce the transport behavior by fully understanding of kinematics of flow field. The eddy-renewal model provides a concept to predict the transport processes by specified turbulent motion under reasonable simplifications. A validation of the model by simulation data which is based on resolving the first principles of fluid dynamics for a wind wave is carried out. The model assumes that the effect of turbulent-convection has the same order of magnitude as the effect of diffusivity in the diffusive sublayer, which is not consistent with the turbulent behaviors observed in the simulation data. This might cause the overestimated heat flux across interface. However, one feature of this model is that it can express the spatial characteristics in transfer processes. The different depths of the bulk field between the upwelling and downwelling regions indicated by the estimated bulk value might shed some light of their roles in transfer processes. Furthermore, it concludes that the length scales of eddies in both of horizontal and vertical directions, and turbulent intensities are the main parameters for transport. This also reveals the possibility to infer the transfer behaviors from spatial characteristics on surface signatures based on the more realistic hydrodynamics background.

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