

## ENSEMBLE OPTICAL FLOW

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For many fluidic applications it is necessary to determine average motion fields rather than instantaneous flow fields (e.g. drag, lift, forces...). Common image motion estimation methods like Particle Image Velocimetry (PIV) [RA07] or Optical Flow (OF) [HS81] are usually applied to estimate instantaneous velocity fields that are later averaged. In order to estimate averaged velocity fields there exists a modified PIV algorithm, *Ensemble PIV*, which does the averaging before the determination of the velocity field [DE99]. This approach is less prone to yield spurious estimates than the general vector averaging technique [ME00]. However, an equivalent ensemble algorithm does not exist for optical flow methods so far. Therefore, we propose an *Ensemble Optical Flow* (EOF) algorithm. This algorithm minimizes a cost functional  $J$  that depends on all  $N$  images  $I^{(t_i)}$  and a regularizing term in order to determine the average velocity field  $\bar{u}$ .

$$\bar{u} = \underset{u}{\operatorname{argmin}} J(u) \quad J(u) = \sum_{i=1}^{N-1} \left\| u^T \nabla I^{(t_i)} + \partial_t I^{(t_i)} \right\|_{L^2(\Omega)}^2 + \frac{\alpha}{2} \left\| \nabla u \right\|_{L^2(\Omega)}^2$$

Note that for  $N = 1$  the algorithm simplifies to the standard Horn & Schunck OF [HS81]. Other than OF, this problem is mathematically well-posed without any regularization ( $\alpha = 0$ ) and is solvable at every pixel of the image. In this case the solution can be calculated explicitly with very low computational effort by using *principle component analysis* (PCA) of image gradient covariance matrices. The PCA is actually the reason for the robustness since outliers are not weighted by their magnitude like it is done in vector averaging.

The EOF algorithm was tested on synthetic images and compared to the results of other state-of-the-art methods. Moreover, we applied EOF to experimental images of an opaque submerged buoyant jet (see Figure 1) [CR08] and submarine images from the *Deepwater Horizon* oil spill (see Figure 2). The results were also compared to various other methods and show that EOF yields better average flow estimates and is more robust against outliers.

Furthermore, the evaluation time lies within a fraction of a second, which makes it interesting for many applications and can allow real-time analysis.

Keywords: optical flow, ensemble average, average flow



Figure 1 Buoyant jet [CR08]



Figure 2 Deepwater Horizon oil spill pipe

## References

- [CR08] TJ Crone, R McDuff, WSD Wilcock: Optical plume velocimetry: a new flow measurement technique for use in seafloor hydrothermal systems, *Exp Fluids* 45:899-915 (2008)
- [DE99] E Delnoij, J Westerweel, N Deen, J Kuipers, W van Swaaij: Ensemble correlation PIV applied to bubble plumes rising in a bubble column. *Chem Eng Sci* 54:5159–5171 (1999)
- [HS81] B Horn B Schunck: Determining optical flow. *Artificial Intelligence* 17:185–203 (1981)
- [ME00] CD Meinhart, S Wereley, J Santiago: A PIV algorithm for estimating time-averaged velocity fields, *J Fluids Eng* 122:285-289 (2000)
- [RA07] M Raffel, C Willert, S Wereley, J Kompenhans: *Particle Image Velocimetry: A practical guide*. Springer Verlag 2nd Edition (2007)